1.3 Mathematical Concepts:

Variables: We use any symbol such as x, y, z, etc. which can assume any specified set of values called Domain. If a variable can be assume only one value is said to be a constant.

Linear function: is a function whose graph is a line in the plane and has the following form:

$$y = a + bx$$

a is a constant and its value equal y when $x = 0$ and b is the slope of the curve or graph plotted between the two variables x and y.

The slope is the rate of change of y with respect to the change of x by one unit:

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Write the equation of a straight line that passes from the following two points $p_1(-1,0)$ and $p_2(1,2)$?

Solution: We calculate the slope b and a

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{1+1} = \frac{2}{2} = 1$$

We find a by $y = a + bx \Longrightarrow 0 = a - 1 \Longrightarrow a = 1$

We can write the equation $y = 1 + x$.

Linear function may have two or more variables as the following:

$$y = a + b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n$$

$$y = a + \sum_{i=1}^{n} b_i\, x_i \qquad (i \text{ runs from } 1 \text{ to } n)$$

Example: Bone mineral density (B M D) depends on age, weight and sex. So, in order to describe the equation that represent the (B M D) we can write:

$$\text{BMD} = a + b_1(\text{age}) + b_2(\text{weight}) + b_3(\text{sex})$$

Nonlinear function: a function with a degree more than one is called nonlinear function. The simplest example to such a function is the **quadratic integer function** which is

$$y = ax^2 + bx + c$$

When $y = 0$, the rate (x) in this case

$$x = \frac{-b \mp \sqrt{b^2 - 4\,a\,c}}{2a}$$

In general, the integer nonlinear function it can be represented by the following:

$$y = a + b_1x^1 + b_2x^2 + b_3x^3 + \cdots + b_nx^n$$

$$y = a + \sum_{i=1}^{n} b_i\, x^i$$

Other functions like exponent $(y = e^x)$ and logarithm $(\log x\ ,\ \ln x)$.

1.4 Statistical Notations:

Notation is an important part of communication in mathematics. Using the correct notation for statistical concepts is essential. BE CAREFUL! In statistics, unlike algebra, you are NOT free to substitute another letter in place of standard notation. Each of the above letters has a specific meaning in statistics.

Note 1: If **Y** or **X** are variables and y_i or x_i are the values of these variables.

Example: Age of 5 students are: (20, 18, 24, 22, 19)

$y_i = 20, 18, 24, 22, 19$ that is $y_1 = 20, y_2 = 18, y_3 = 24, y_4 = 22, y_5 = 19$

Where $i = 1,2,3, \dots, n$ here $n = 5$

1.4.1 Summation Notation

$\sum_{i=1}^{n} x_i$ mean $x_1+x_2+x_3+\dots+x_n$, where the letter i is called the index of summation

Note: i is dummy variable and it can be replaced any letter

that is $x_1+x_2+x_3+\dots+x_n = \sum_{i=1}^{n} x_i = \sum_{j=1}^{n} x_j = \sum_{k=1}^{n} x_k$

from above example

$$\therefore \sum_{i=1}^{5} y_i = y_1 + y_2 + y_3 + y_4 + y_5$$

$$\sum_{i=1}^{n=5} y_i = 20 + 18 + 24 + 22 + 19 = 103$$

$$\sum_{i=1}^{n=5} y_i^2 = (20)^2 + (18)^2 + (24)^2 + (22)^2 + (19)^2 = 400 + 324 + 576 + 484 + 361 = 2145$$

$$\left(\sum_{i=1}^{n=5} y_i\right)^2 = (20 + 18 + 24 + 22 + 19)^2 = (103)^2 = 10609$$

We note that

$$\sum y_i^2 \neq \left(\sum y_i\right)^2$$

Example: If we have two variables such as:

$$x_i = 2, 4, 6, 8, 10 \qquad \text{and} \qquad y_i = 1, 3, 5, 7, 9$$

We can compute all measurements as the following:

$$\sum x_i , \sum y_i , \sum x_i\, y_i, \sum x_i^2 , \sum y_i^2 , \sum x_i^2 * \sum y_i^2$$

$$,\left(\sum x_i\right)^2, \left(\sum y_i\right)^2, \left(\sum x_i\, y_i\right)^2, \left(\sum x_i\right)^2, \left(\sum y_i\right)^2$$

$$\sum x_i = 2 + 4 + 6 + 8 + 10 = 30$$

$$\sum y_i = 1 + 3 + 5 + 7 + 9 = 25$$

$$\sum x_i\, y_i = 2 * 1 + 4 * 3 + 6 * 5 + 8 * 7 + 10 * 9 = 2 + 12 + 30 + 56 + 90 = 190$$

$$\sum x_i * \sum y_i = 30 * 25 = 750$$

$$\sum x_i^2 = 4 + 16 + 36 + 64 + 100 = 220$$

$$\sum y_i^2 = 1 + 9 + 25 + 49 + 81 = 165$$

$$\sum x_i^2 * \sum y_i^2 = 220 * 165 = 36300$$

$$\sum x_i^2 * y_i^2 = 4 * 1 + 16 * 9 + 36 * 25 + 64 * 49 + 100 * 81$$

$$= 4 + 144 + 700 + 3136 + 8100 = 12284$$

$$\left(\sum x_i\right)^2 = (30)^2 = 900$$

$$\left(\sum y_i\right)^2 = (25)^2 = 625$$

$$\left(\sum x_i\right)^2 * \left(\sum y_i\right)^2 = 900 * 625 = 562500$$

Note 2:

$$\sum x_i\, y_i \neq \sum x_i * \sum y_i$$

$$\sum x_i^2 * \sum y_i^2 \neq \left(\sum x_i\, y_i\right)^2$$

We can summarize all the above calculations in a table form, see Table 1.

Table 1: Some mathematical operations on x_i and y_i.

x_i	y_i	x_i^2	y_i^2	$x_i y_i$	$x_i^2 * y_i^2$
2	1	4	1	2	4
4	3	16	9	12	144
6	5	36	25	30	900
8	7	64	49	56	3136
10	9	100	81	90	8100
$\sum x_i = 30$	$\sum y_i = 25$	$\sum x_i^2 = 220$	$\sum y_i^2 = 165$	$\sum x_i\, y_i = 190$	$\sum x_i^2 * y_i^2 = 12284$

Note 3: If c is a constant, we can write

$$\sum_{i=1}^{n} c_i = (c_1 + c_2 + c_3 + \cdots + c_n) = n * c$$

$$\sum_{i=1}^{n} 2 = (2 + 2 + 2 + \cdots + 2_n) = n * 2$$

Now if

$$\sum cy_i = (cy_1 + cy_2 + cy_3 + \cdots + cy_n) = c(y_1 + y_2 + y_3 + \cdots + y_n) = c \sum y_i$$

If $c = 3$, then

$$\sum 3y_i = (3y_1 + 3y_2 + 3y_3 + \cdots + 3y_n) = 3(y_1 + y_2 + y_3 + \cdots + y_n) = 3 \sum y_i$$

Note 4:

$$\sum (x_i + y_i) = \sum x_i + \sum y_i$$

If $x_i = 2, 4, 6, 8, 10$ and $y_i = 1, 3, 5, 7, 9$

$$\sum (x_i + y_i) = (2 + 1) + (4 + 3) + (6 + 5) + (8 + 7) + (10 + 9) = 55$$

$$\sum x_i = 30, \qquad \sum y_i = 25$$

$$\sum x_i + \sum y_i = 30 + 25 = 55$$

Note 5:

$$\sum (x_i + y_i)^2 = \sum x_i^2 + 2 \sum x_i y_i + \sum y_i^2$$

$$= 220 + 380 + 165 = 765$$

$$\sum (x_i + y_i)^2 = (2 + 1)^2 + (4 + 3)^2 + (6 + 5)^2 + (8 + 7)^2 + (10 + 9)^2 = 765$$

Note 6:

$$\sum x_i / y_i \neq \sum x_i / \sum y_i$$

$$\sum x_i / y_i = \left(\frac{x_1}{y_1} + \frac{x_2}{y_2} + \frac{x_3}{y_3} + \cdots + \frac{x_n}{y_n}\right) = \frac{2}{1} + \frac{4}{3} + \frac{6}{5} + \frac{8}{7} + \frac{10}{9}$$

$$= 2 + 1.3 + 1.2 + 1.14 + 1.11 = 6.75$$

$$\sum x_i / \sum y_i = (2+4+6+8+10) \,/\, (1+3+5+7+9) = 30/25 = 1.2$$

Note 7:

Example: If $x_i = 5, 7, 9, 11$

$$\sum (x_i - 3) = (5-3) + (7-3) + (9-3) + (11-3) = 2+4+6+8 = 20$$

$$\sum (x_i) = 5+7+9+11 = 32 \qquad n*3 = 4*3 = 12$$

$$\sum (x_i - 3) = \sum x_i - n*3 = 32 - 12 = 20$$

$$\sum (x_i^2 - 3) = (25-3) + (49-3) + (81-3) + (121-3) = 264$$

$$\sum x_i^2 = 25+49+81+121 = 276 \qquad n*3 = 4*3 = 12$$

$$\left(\sum x_i^2\right) - n*3 = 276 - 12 = 264$$

$$\therefore \sum (x_i^2 - 3) = \left(\sum x_i^2\right) - n*3$$

Example: If $x_i = 5, 7, 9, 11,$ Find $\sum (x_i - 3)^2$

Solution:

$$\sum (x_i - 3)^2 = \sum (x_i^2 - 2*3x_i + n*3^2) = \sum x_i^2 - 6*\sum x_i + 4*9$$

$$276 - 6*32 + 36 = 312 - 192 = 120$$

$$\sum (x_i - 3)^2 = (5-3)^2 + (7-3)^2 + (9-3)^2 + (11-3)^2 = 4+16+36+64 = 120$$

Some important sums

$$\sum_{i=1}^{n} \boldsymbol{i} = \mathbf{1+2+3+4}+\cdots+\boldsymbol{n} = \frac{\boldsymbol{n(n+1)}}{\mathbf{2}}$$

$$\sum_{i=1}^{n} \boldsymbol{i^2} = \mathbf{1^2+2^2+3^2}+\cdots+\boldsymbol{n^2} = \frac{\boldsymbol{n(n+1)(2n+1)}}{\mathbf{6}}$$

$$\sum_{i=1}^{n} \boldsymbol{i^3} = \mathbf{1^3+2^3+3^3}+\cdots+\boldsymbol{n^3} = \left(\frac{\boldsymbol{n(n+1)}}{\mathbf{2}}\right)^{\mathbf{2}}$$

H.W

1- Simplify

$$\sum_{i=1}^{n}(i-1)^2$$

2- If $y_i=3x_i+2$, $i=1,2,3,4,5$ *and* $\sum_{i=1}^{5} xi = 20$

Find $\sum_{i=1}^{5} yi$